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ABSTRACT A new model of the height profile of tropospheric refractivity is presented as the basis for an improved tropospheric correction for satellite doppler or range data. The model treats the "dry" and "wet" components of the refractivity separately, each as a fourth-degree function of height above the geoid. The height parameters are different for the two and are obtained from meteorological balloon data. A latitude dependence has been found for the dry height. The resulting expressions for the correction of satellite doppler and range data are presented, and their effectiveness is shown by figures that give two different kinds of observed data: doppler residuals for several satellite passes, and the navigation error in station-to-orbit slant range. In each case the data are given without and with the correction. The use of the correction removed obvious systematic errors.

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SEPTEMBER 1968

Technical Memorandum

A TWO-QUARTIC REFRACTIVITY PROFILE FOR THE TROPOSPHERE, FOR CORRECTING SATELLITE DATA

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ABSTRACT

A new model of the height profile of tropospheric refractivity N is presented as the basis for an improved tropospheric correction for satellite doppler or range data. ($N \equiv 10^6 (n-1)$, where n is the index of refraction.) The model treats the "dry" and "wet" components of N separately, and represents each as a fourth-degree function of height above the geoid; each component profile starts with its locally observed surface value and decreases to zero at an effective height which is different for the two components. The height parameters were obtained by a least squares fit to observed data. A latitude dependence has been found for the "dry" height. The model was found capable of closely matching any local average N profile observed in a world-wide sample of locations, throughout the height range of meteorological balloon data (up to 24 km); samples are shown. The resulting expressions for the correction of satellite doppler and range data are presented; they are finite and usable at all elevation angles. The effectiveness of the correction is evidenced by figures showing two different kinds of observed data: first, doppler residuals for several satellite passes, respectively <u>without</u> and <u>with</u> the use of the correction; and second, the "navigation" error in station-to-orbit slant range from doppler data, again <u>without</u> and <u>with</u> the correction. The use of the correction removed obvious systematic errors. The fact that satellite doppler data display identifiable tropospheric effects is of interest with regard to future study of the troposphere.

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I. Introduction

The doppler shift of a radio signal from a satellite, as received at known tracking station positions, is used in the Navy navigational satellite system to determine the orbit of the satellite; the position of any other observing station can then be determined when the orbit is known [Guier and Weiffenbach, 1960].

The apparent (electromagnetic) range to the satellite from a tracking station is $\int n ds$ along the signal path; n being the index of refraction of the medium, varying with position along the path. The observed doppler shift of the satellite signal, neglecting non-atmospheric errors, is then

$$\Delta f_{obs} = -\frac{f}{c}\frac{d}{dt}\int n ds \qquad (1)$$

where f is the signal frequency and c the vacuum velocity of light. But the "vacuum" doppler shift is needed for computing the orbit:

$$\Delta f_{v} = -\frac{f}{c}\frac{d\rho}{dt} \qquad (2)$$

where ρ is the geometric slant range to the satellite.

Both ionospheric and tropospheric effects must be removed from the observed doppler to get the vacuum doppler shift. The two-frequency method [Guier and Weiffenbach, 1960] which removes first-order ionospheric effects does not remove the effects of the troposphere, whose refractive

index is independent of frequency up to approximately 15000 Mhz. A theoretical tropospheric correction is therefore required and must be based on some model of the troposphere; the term "troposphere" will be used here to include all the lower, uncharged part of the atmosphere.

An initial tropospheric model and a subsequent improvement [Hopfield, 1963, 1965], which will here be designated as Model I and Model II, have been superseded by the model to be described below.

II. Assumptions and Requirements for a Troposphere Model

It will be assumed, as before, that the refractive index of the troposphere is a function of its value at the surface and of height above the earth, but not of horizontal distance or time, within the time and space pertinent to a satellite pass. The curvature of the signal path affects doppler data only if it is great enough to affect the path length perceptibly, therefore only near the horizon. If this effect is neglected, the integral of equation (1) can be taken along the slant range vector, like that of equation (2). For ionosphere-corrected data containing only atmospheric errors, the difference between these two is the tropospheric effect. This difference, expressed in terms of the refractivity N, is

$$\Delta f_{tro} = -10^{-6} \frac{f}{c} \frac{d}{dt} \int N \, d\rho \qquad (3)$$

Effectively, the integral now extends only through the troposphere, since N becomes zero at the "top" of the troposphere. Expression (3) provides

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the needed tropospheric doppler correction, given an adequate expression for N as a function of the height h ; i.e., an adequate N profile.

To give the proper correction throughout a satellite pass, a theoretical N profile must yield the observed value of $\int N d\rho$ through the troposphere at all elevation angles which are to be used. To do this at high angles, the zenith value, $\int N dh$, must be correct. To succeed at low angles, the theoretical N profile must also match the observed profile in shape near the surface of the earth. Since low angle satellite data can often be discarded, the former of these two conditions (the zenith integral) is the more essential one for satellite purposes,* and is the condition on which Models I and II of the correction were based. Both of those versions were expected to be less satisfactory at low angles (e.g., below 10°) than at high angles. However, both versions made equation (3) integrable in closed form and both could be used even at the horizon, though with some loss of accuracy.

To obtain an observed N profile for comparison purposes, the values of N at different heights were computed from a set of meteorological balloon data [Monthly Climatic Data for the World], using the equation of Smith and Weintraub [1953] at each observed data point:

$$N = \frac{77.6}{T_K} \left(P + \frac{4810\ e}{T_K} \right) \tag{4}$$

In this, T_K is temperature (degrees Kelvin), P is total atmospheric pressure and e the partial pressure of water vapor (both in millibars).

*A similar condition for the problem of astronomical refraction was recognized almost two centuries ago by Oriani [Mahan, 1962]

The area under the resulting N vs h profile then provides the observed value of the zenith integral $\int N dh$ for the set of observed data. The theoretical profile to be compared with this is based on the same surface value of N , i.e., the one provided by the observed data.

Model II of the theoretical correction succeeded in matching a world-wide sampling of observed values of $\int N dh$ generally with an accuracy of 2 or 3 percent (using a monthly average N profile in each case). This version, therefore, was expected to be satisfactory except at "low" angles, the definition of "low" being somewhat uncertain.

To clarify this point and to evaluate the correction, some station positions were determined from satellite doppler data, respectively <u>without</u> and <u>with</u> the use of the Model II tropospheric correction. Refraction by the troposphere steepens the slope of the doppler shift vs. time curve as observed during a satellite pass. If no horizontal gradient of N is present and if tracking data are symmetrical about the point of closest approach, the troposphere affects only the slant range determination (tracking station to orbit), making the station appear closer to the orbit than it actually is. This effect is a function of pass geometry; it averages approximately 20 meters for a high-angle pass, but 100 meters for a pass whose maximum elevation is only 10°. (An illustration of the effect will be shown later.) The use of the above correction removed this effect from high-angle passes, on the average, as far as could be detected; but systematically over-corrected by approximately 10 percent in low-angle passes. When used for a 10° pass, it made the average station appear 10 or 15 meters

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too far away from the orbit. This low angle inaccuracy called for an improvement in the N profile shape.

III. Matching the Shape of the Observed N Profile: a Two-Quartic Expression

The desired theoretical N profile would start with the observed N at the surface and yield not only the correct zenith integral $\int N dh$ but also, if possible, the correct lapse rate $\frac{dN}{dh}$ near the surface of the earth, at any geographical location. Some expressions which were tried and discarded will be mentioned briefly.

The well-known CRPL exponential atmosphere [Bean and Thayer, 1959] gives, to a good approximation, the observed lapse rate of N near the surface. It was designed to do this especially in the continental U. S. and was intended for low-angle applications, especially within the troposphere. It was not designed specifically for satellite applications and its zenith integral does not match the observed one for all climates. If applied to marine or other humid climates, it is likely to make this integral too small, often by 10 or 15 percent. In addition, it is inconvenient for use with doppler data. Since the integral $\int N d\rho$ for exponential N , except at the zenith, is not integrable in closed form and requires approximation [Freeman, 1962], the evaluation of its time derivative for equation (3) also entails approximations; their validity is a function of the elevation angle. A model more suited to satellite data was therefore desired.

In contrast, any N profile of the form

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$$N = k\,(h_{o_\mu} - h)^\mu \quad , \quad h \le h_{o_\mu} \qquad --- \qquad (5)$$

will make equation (3) integrable; μ here is a positive integer and h_{o_μ} the corresponding equivalent height of the troposphere. A height h_{o_μ} can easily be chosen for any value of μ, to yield the correct value of $\int N dh$, if a correction factor is used as for Model II; but such a profile will not in general match the observed N profile near the earth. A simple profile of the form of equation (5), therefore, was also ruled out.

Separate treatment of the so-called "dry term" and "wet term" of the refractivity was then considered. These are the components of N in equation (4):

$$\left.\begin{aligned} N_d &= \frac{77.6\ P}{T_K} \\ N_w &= \frac{77.6\ (4810\ e)}{T_K^{\,2}} \end{aligned}\right\} \qquad --- \qquad (6)$$

where d and w refer to "dry" and "wet" respectively. Even a cursory look at upper air meteorological data [Monthly Climatic Data for the World] shows that at heights of 9 or 10 km, where atmospheric pressure is still at nearly one-third of its surface value, the moisture content is nearly zero (barely, if at all, measurable), even in tropical latitudes. The components N_d and N_w are therefore different functions of height.

Evidence that only a part of the tropospheric range or range rate error is weather dependent had been found in developing the Model II

correction [Hopfield, 1965]. A two-part exponential treatment of the profile had already been suggested by B. R. Bean [1961], with especial reference to the bending of a low-angle radio ray in tropospheric propagation. As regards satellite use, however, the bi-exponential has the same disadvantages as a simple exponential profile and was therefore not adopted. The recent three-parameter exponential treatment [Bean et al, 1966] could provide the proper area under the N profile; but with added problems in evaluating the correction. It has not been used here.

It was found practical, for satellite doppler or range data, to employ a two-part treatment which uses two functions of the form of equation (5), with $\mu = 4$ for each part. The resulting "dry" profile is

$$N_d = k_d (h_{o_d} - h)^4 \quad , \qquad h \le h_{o_d}$$

If h_{o_d} is chosen to give the observed area under a "dry" profile and the value of k_d is determined from this and the surface conditions, the resulting N_d profile gives a good approximation to the shape of the observed profile up to the usual height of balloon data, even the lapse rate near the surface ordinarily being within a few percent of the observed value. The value of h_{o_d} is of the order of 40 km and will be discussed below.

A quartic equation is also a reasonably good approximation to the usual decrease of N_w with height, i.e.,

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$$N_w = k_w (h_{o_w} - h)^4 \quad , \quad h \le h_{o_w}$$

the value of k_w being likewise obtained from surface conditions and a known or postulated h_{o_w} ; the latter is of the order of 12 km and will also be discussed below.

The total theoretical N profile is then the sum of these two components, and the complete expression becomes:

$$\left.\begin{aligned} N &= \sum_{i=1,2} N_i \\ N_i &= \frac{N_{T_i}}{(h_{o_i} - h_T)^4} (h_{o_i} - h)^4 \qquad && h \le h_{o_i} \\ N_i &= 0 && h > h_{o_i} \end{aligned}\right\} \text{--- (7)}$$

where $i = 1,2$ refers to the "dry" and "wet" components respectively. All heights are measured above the geoid. The subscript T refers to the tracking station.

Samples of observed N profile <u>components</u> (dry and wet), and quartic profile components designed to match them in total area, are shown in Figures 1(a), 2(a), 3(a) and 4(a) for quite different latitudes and climates. They are arranged in order of latitude, heading northward. The <u>total</u> profiles are given in the (b) curves of these figures, the CRPL exponential profile being also shown in each case for comparison. The shape

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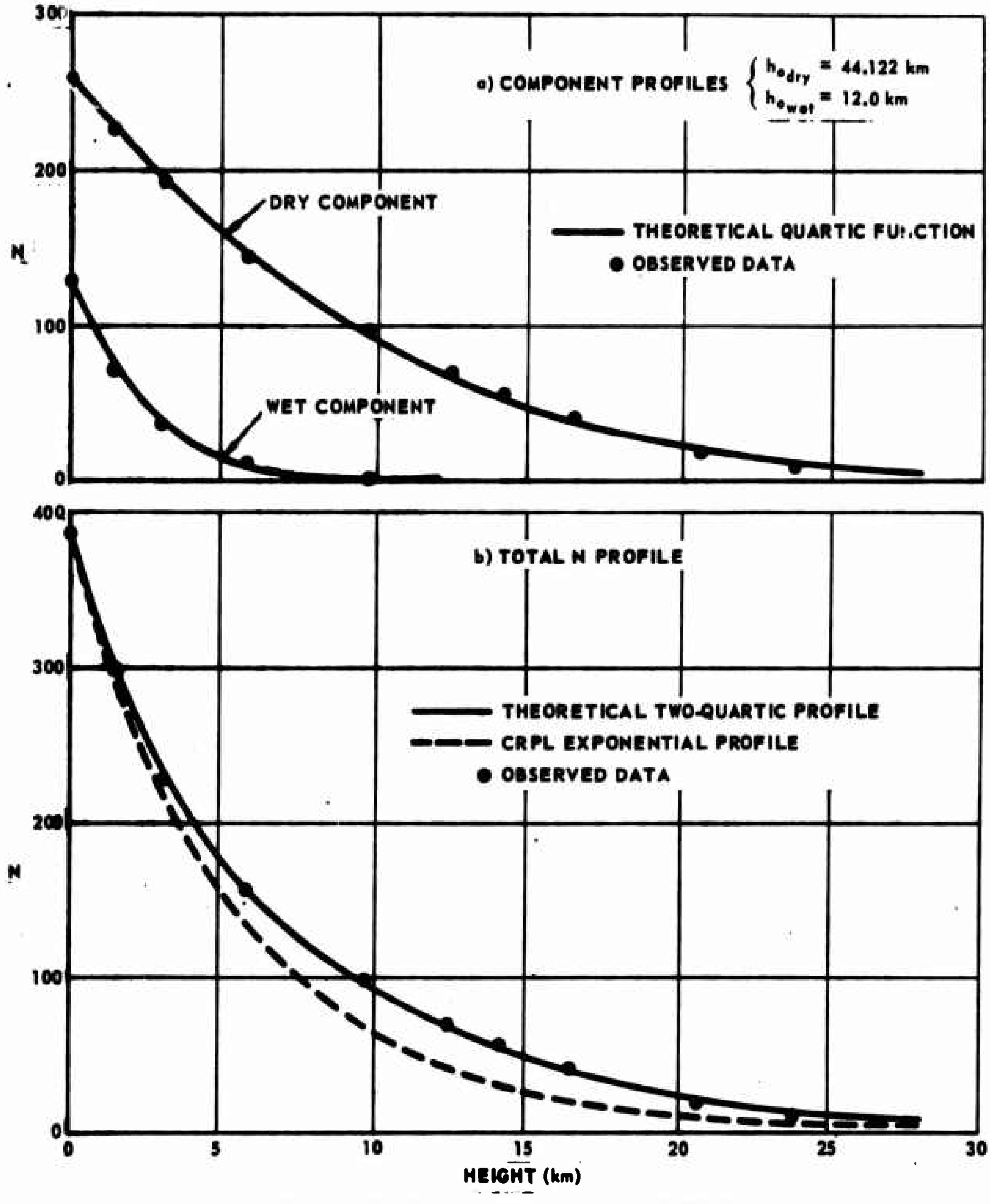

Fig. 1 PROFILE OF TROPOSPHERIC REFRACTIVITY N, CANTON ISLAND, LATITUDE 3° SOUTH – JANUARY 1964

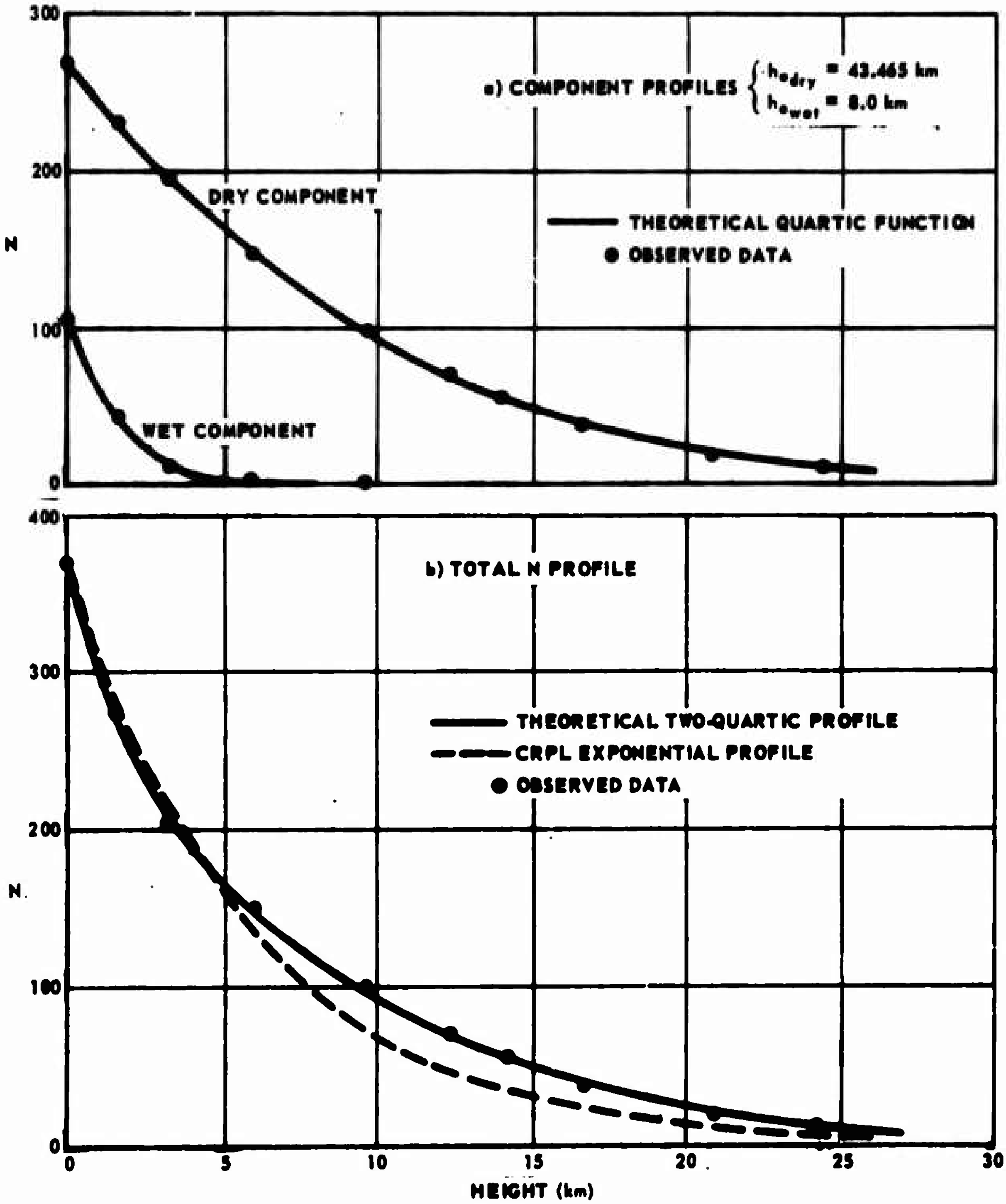

Fig. 2 PROFILE OF TROPOSPHERIC REFRACTIVITY N, WEATHER SHIP E, LATITUDE 35° NORTH – JULY 1967

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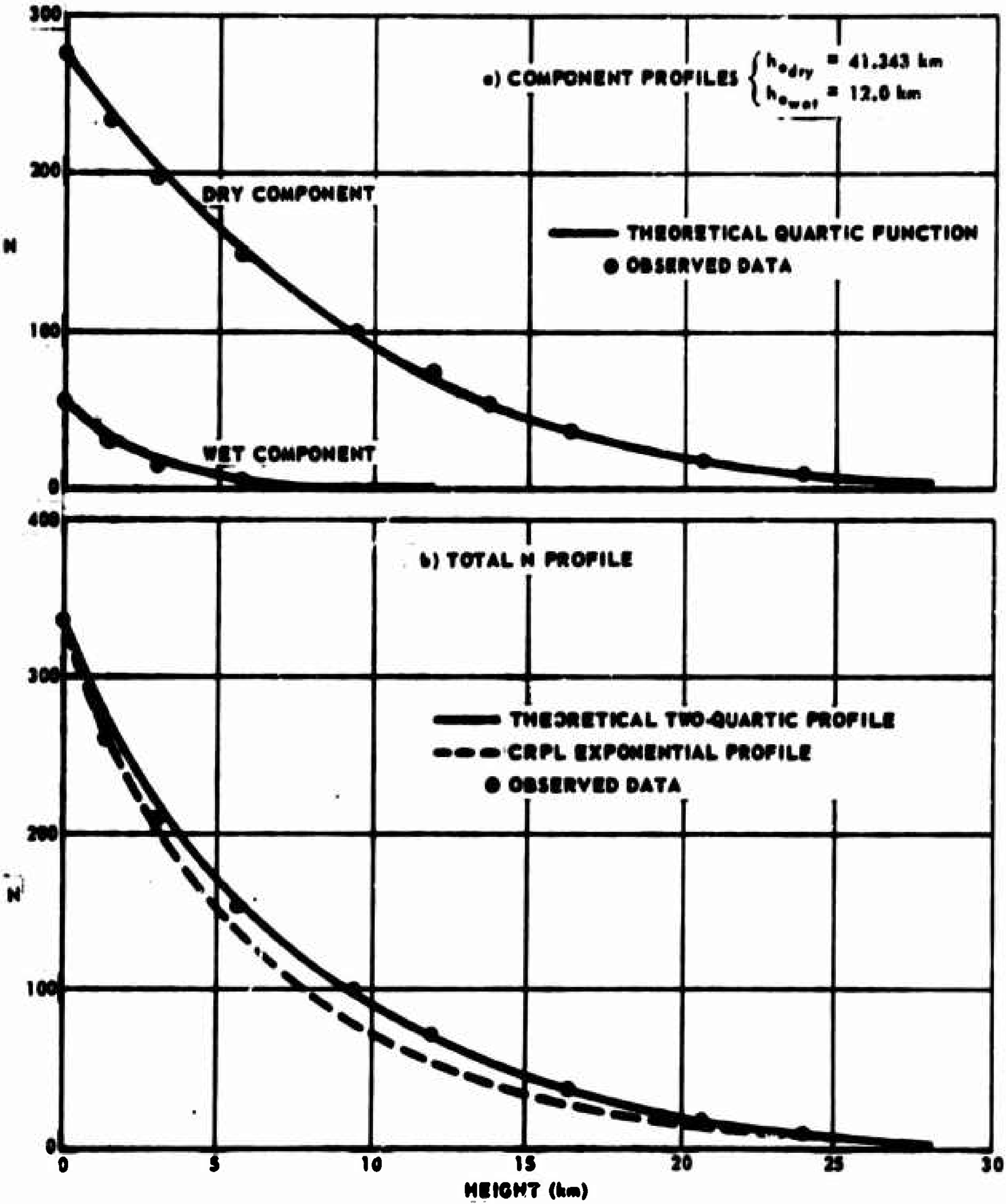

Fig. 3 PROFILE OF TROPOSPHERIC REFRACTIVITY N, WASHINGTON, D.C., LATITUDE 39° NORTH – MAY 1964

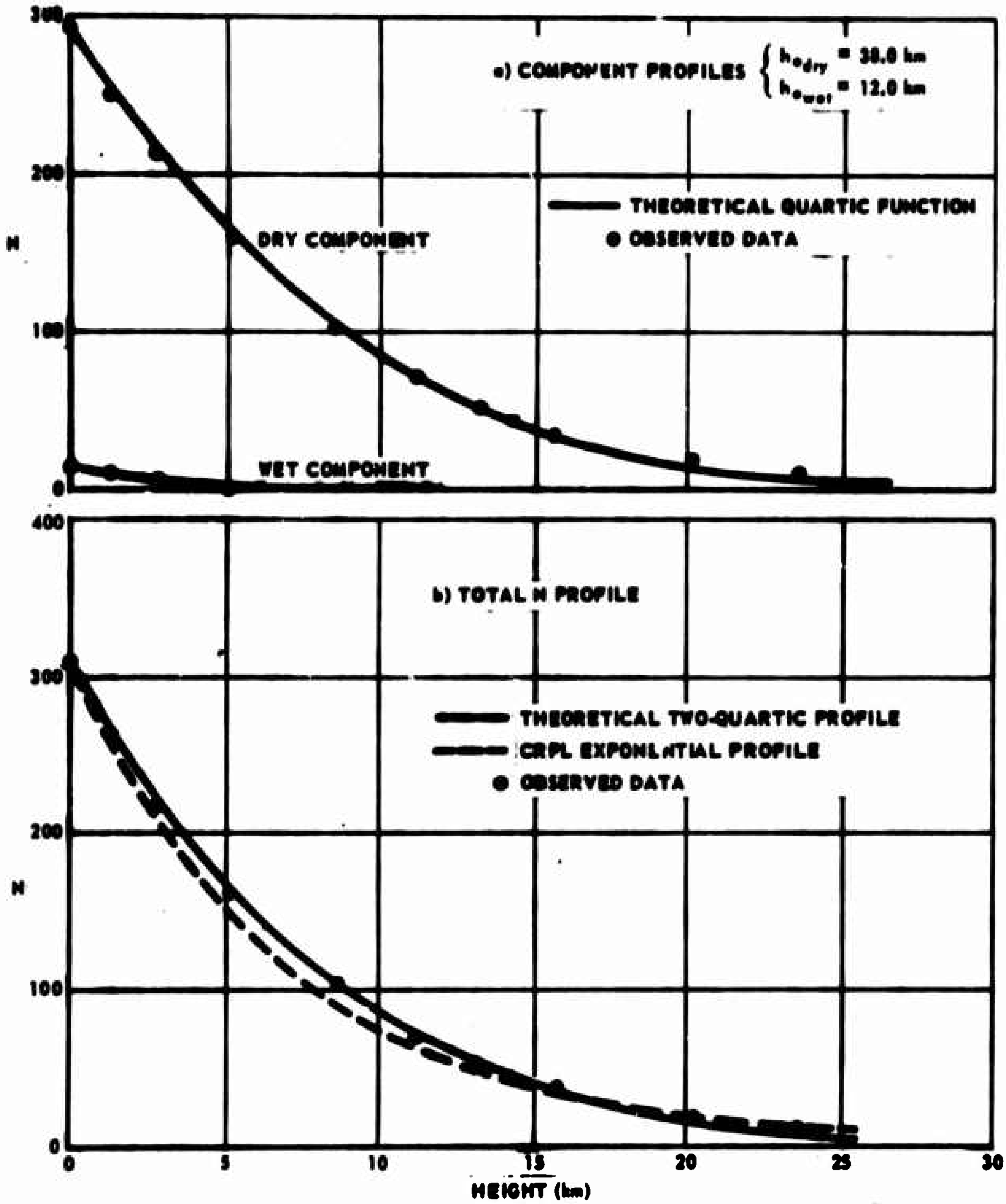

Fig. 4 PROFILE OF TROPOSPHERIC REFRACTIVITY N, ANCHORAGE, ALASKA, LATITUDE 61° NORTH – FEBRUARY 1964

of the two-quartic expression is a good approximation to the observed data in all cases. The area under the exponential profile is too small by only 3 percent for the Anchorage, Alaska profile (Figure 4), but by 17 percent for the moist tropical climate of Canton Island (Figure 1).

Observed N_d profiles are regular in form and differ from each other to a relatively minor extent, while observed N_w profiles may be irregular and differ widely in different localities and seasons, or even from day to day. This variation will be discussed in a later section.

IV. Height Parameters

The heights h_{o_d} and h_{o_w} of Figures 1, 2, 3 and 4, selected individually for profile area-matching, are not the same. Before the new N model can be used routinely to compute a doppler correction, information is needed on height parameters for both "dry" and "wet" terms for any location.

Balloon data show that the height of any constant pressure surface in the atmosphere is to some extent a function of latitude. The effective height (for refraction) of the uncharged part of the atmosphere is probably also a function of latitude, and an equatorial bulge is likely.

Initially, latitude variations of both h_{o_d} and h_{o_w} were postulated. An attempt was made to find values of h_{o_d} and h_{o_w} at the equator, and the latitude variation of each, using a least squares procedure to match theoretical to observed values of $\int Ndh$. The initial results were questionable, and a different approach was tried, as follows.

The simpler assumption was made as a first approximation, that the height h_{o_w} is invariant with latitude; but that h_{o_d} at a tracking

station latitude φ_T is given by

$$h_{o_d} = h_{o_{d(eq)}} + A_d \sin^2 \varphi_T \tag{8}$$

where $h_{o_{d(eq)}}$ is the "dry" height at the equator and A_d the amplitude of variation of h_{o_d} with latitude. If h_{o_d} has an equatorial bulge, A_d must of course be negative. Integrating equations (7) through the troposphere, the total area under the two-quartic profile is

$$\int_o^{h_{o_d}} N dh = \frac{1}{5} [N_{T_d} (h_{o_d} - h_T) + N_{T_w} (h_{o_w} - h_T)] \tag{9}$$

Using the value of h_{o_d} from equation (8), and a postulated value of h_{o_w}, in equation (9), and equating the resulting theoretical $\int N dh$ to a corresponding observed $\int N dh$ from balloon data (as described above), the unknown parameters $h_{o_{d(eq)}}$ and A_d were obtained by a least squares procedure. The computation was repeated several times with different postulated values of h_{o_w}. The data set included widely varying climates (e.g., Alaska, Algeria, Antarctica), and station heights from sea level to 1500 meters.

The resulting parameters are tabulated below for a few values of h_{o_w}, along with the polar values of h_{o_d} which result from these parameters.

TABLE I

Height Parameters for Two-Quartic N Profile (km)

h_{o_w}	$h_{o_{d(eq)}}$	A_d	Polar value of h_{o_d}
10	43.858	-5.986	37.872
12	43.130	-5.206	37.924
14	42.402	-4.426	37.976

Interestingly, the polar value of h_{o_d} is nearly the same for any of these values of h_{o_w}. The moisture content of the air is so small in polar regions that the solution for the polar h_{o_d} is not much affected by assumptions about h_{o_w}.

The residual errors in $\int Ndh$ (theoretical minus observed value) have an rms value of approximately 2 percent for any of these sets of parameters, i.e., for h_{o_w} ranging from 10 to 14 or even to 16 km. Thus the area ($\int Ndh$) alone is not a sufficient criterion for deciding between the several sets of parameters. Comparison of the shape of a variety of observed with theoretical N profiles (from equations (7)) indicates that a value of h_{o_w} = 12 km, though not correct in all cases (e.g., Figure 2), is generally satisfactory and is suitable for preliminary use. From Table I, we then have

$$h_{o_d} = 43.130 - 5.206 \sin^2 \varphi_T \qquad \text{km.} \qquad (10)$$

at a latitude φ_T .

The ratio of theoretical to observed $\int N dh$ for this model is plotted in Figure 5 as a function of total surface refractivity. The new N model makes this ratio essentially unity for any value of surface refractivity, and unlike Model II, needs no correction factor.

The same ratio is plotted against station latitude in Figure 6. Any systematic departure from a ratio of unity is small enough to be in doubt. It is possible, however, that slightly different height parameters should be used for the northern and southern hemispheres (in analogy to the asymmetry of the earth's gravity field). No further work has been done on this as yet.

Some other possible systematic errors will be mentioned later.

V. Tropospheric Contribution to Satellite Doppler or Range Data

A. Theoretical Expressions

It follows from equations (1) to (3) and the assumptions about the troposphere that the tropospheric contribution to the doppler shift of a satellite signal at any instant is

$$\Delta f_{tro} = - \frac{f}{c} \frac{\partial (\Delta \rho_{tro})}{\partial E} \dot{E} \qquad (11)$$

where E is the elevation angle of the slant range vector to the satellite at the tracking station and $\dot{E}$ its time derivative [Hopfield, 1963].

Using the procedure developed for the Model I correction, but the

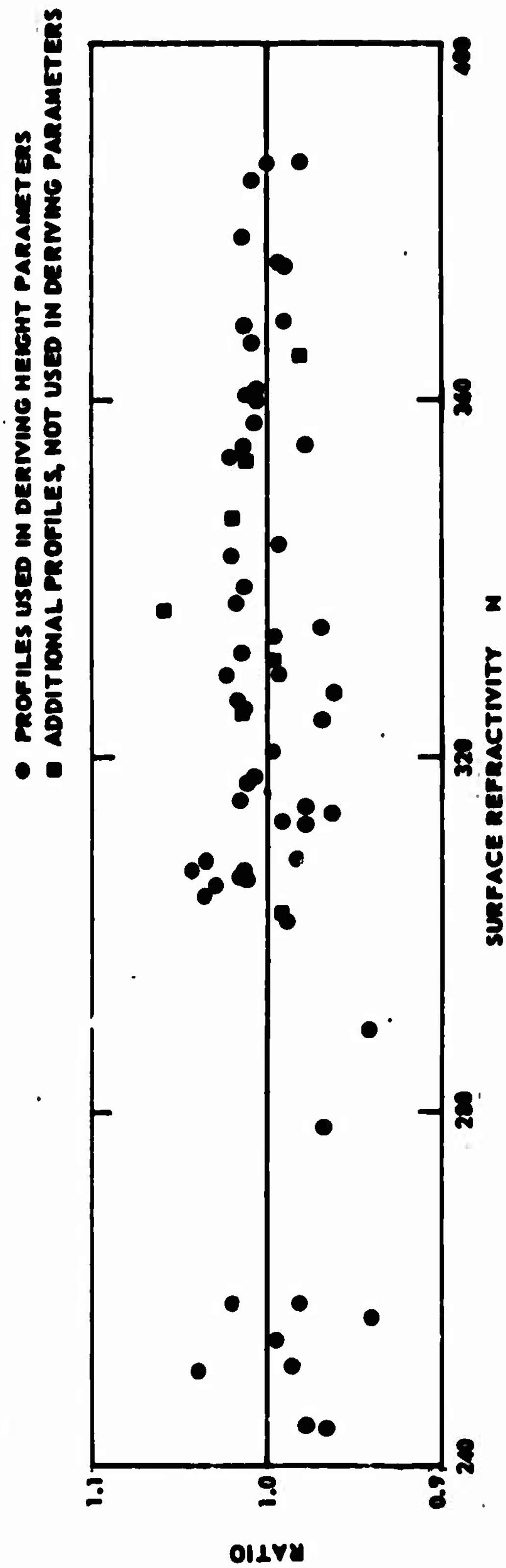

Fig. 5 RATIO OF THEORETICAL TO OBSERVED $\int N dh$ VERSUS SURFACE REFRACTIVITY N

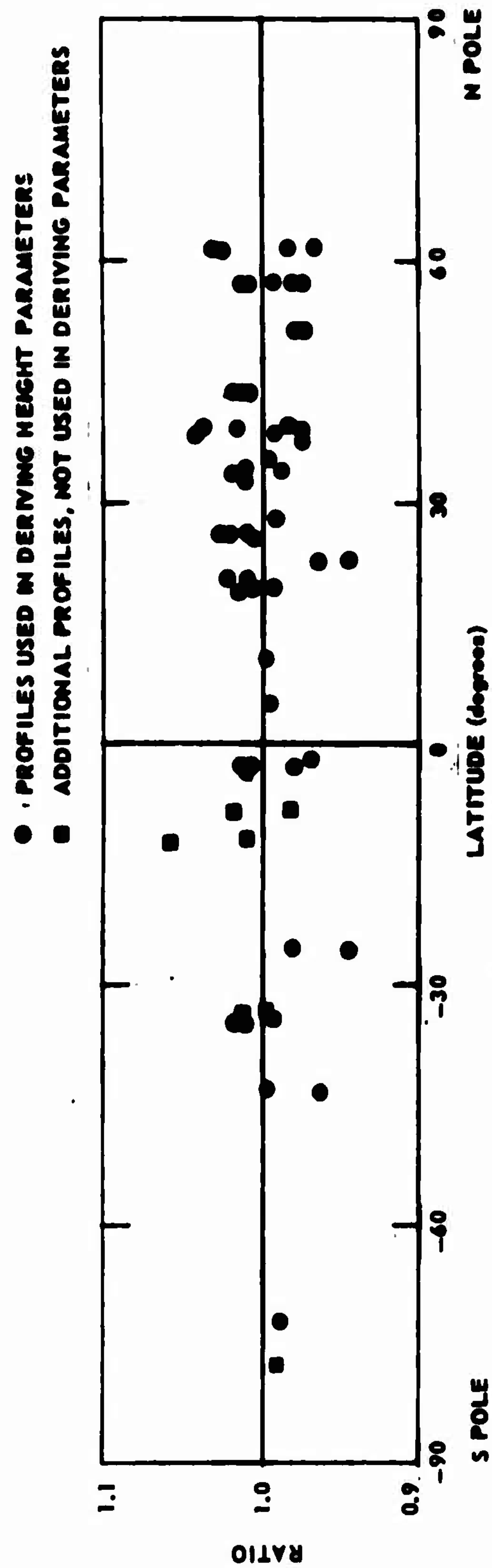

Fig. 6 RATIO OF THEORETICAL TO OBSERVED $\int N dh$ VERSUS STATION LATITUDE

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N profile of equations (7), the tropospheric doppler effect becomes

$$\Delta f_{tro} = 10^{-6} \frac{f}{c} r_T \dot{E} \sum_{i=1,2} \{N_{T_i} F_{4_i} (E) \quad --- \tag{12}$$

where the function $F_{4_i}(E)$ for either component (subscripted "4" to refer to the quartic N profile) is

$$\left. \begin{aligned} F_{4_i}(E) = \cos E \Bigg[1 + \frac{4\ell_1}{h^4_{tro_i}} \Bigg\{ & \frac{\ell^3_{3_i} - \ell_1^{\,3}}{3} + \ell_{3_i} (\ell_2^{\,2} + \frac{3 r^2_{tro_i}}{2}) \\ & - \ell_1 (\ell_2^{\,2} - \frac{3 r_T r_{tro_i}}{2} + 3 r^2_{tro_i}) \\ & + (\frac{3 r_{tro_i} \ell_2^{\,2}}{2} + r^3_{tro_i}) \ln \frac{r_T + \ell_1}{r_{tro_i} + \ell_{3_i}} \Bigg\} \Bigg] \end{aligned} \right\} \tag{13}$$

In this expression, r_T and r_{tro_i} are distances from the center of the earth respectively to the tracking station and to the "top" of the troposphere (dry or wet component). Also

$$h_{tro_i} = h_{o_i} - h_T$$

$$r_{tro_i} = r_T + h_{tro_i}$$

$$\ell_1 = r_T \sin E$$

$$\ell_2 = r_T \cos E$$

$$\ell_{3_i} = (r^2_{tro_i} - \ell_2^{\,2})^{1/2}$$

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The value of $F_{4_i}(E)$ is obviously zero at $E = 90^\circ$ and unity at $E = 0^\circ$ for any station height and troposphere height, for both "dry" and "wet" terms.

If the tropospheric range contribution rather than a doppler contribution is desired (as in using radar or integrated doppler data), this is obtained by evaluating $\int(n-1)d\rho$ with the same input information which has just been described. The range effect (assuming no ray bending) is given by the sum of the "dry" and "wet" contributions:

$$\Delta\rho_{tro} = \sum_{i=1,2} \Delta\rho_i$$

where

$$\Delta\rho_i = 10^{-6} N_{T_i} \Bigg[-\ell_1 + \frac{4}{h_{tro_i}^4} \Bigg\{ \frac{1}{3} r_T^2 \ell_1^3 - \frac{2}{15} \ell_1^5 - \frac{3}{4} r_T r_{tro_i} \ell_1 (\ell_1^2 + \frac{1}{2} \ell_2^2)$$

$$+ r_{tro_i}^2 \ell_1^3 - \frac{1}{2} r_{tro_i}^3 r_T \ell_1 - \frac{1}{3} r_{tro_i}^2 \ell_{3_i}^3 + \frac{2}{15} \ell_{3_i}^5$$

$$+ \frac{3}{4} r_{tro_i}^2 (\ell_{3_i}^3 + \frac{1}{2} \ell_{3_i} \ell_2^2) - r_{tro_i}^2 \ell_{3_i} (\ell_{3_i}^2 - \frac{1}{2} r_{tro_i}^2)$$

$$+ \frac{1}{2} r_{tro_i} \ell_2^2 (\frac{3}{4} \ell_2^2 + r_{tro_i}^2) \ln \frac{r_T + \ell_1}{r_{tro_i} + \ell_{3_i}} \Bigg\} \Bigg]$$

In practice, using an IBM 7094 computer, the computation of $F_{4_i}(E)$ or $\Delta\rho_i$ is done in double precision to avoid excessive rounding errors. To

save computing time, a table of $F_{4_i}(E)$ vs E can be computed for each satellite pass, since r_T and r_{tro_i} pertain to a particular station; the value of $F_{4_i}(E)$ for a specified elevation angle can then be obtained by interpolating in the table.

If a tropospheric range rate effect is desired, this is obtained directly from equations (12) and (13) by deleting the scaling factor $\frac{f}{c}$ in equation (12).

Equations (12), (13) and (14) come from exact integral expressions. They are based on physical approximations but contain no mathematical approximations. To the extent of validity of the physical assumptions, they are therefore good at all elevation angles, and do in fact remain finite at all angles. The tropospheric contribution to range rate for a moderately high pass, obtained from equation (12), is of the order of 2 meters/sec at the geometrical horizon, where the actual range rate is approximately 6 km/sec, for satellites 1000 km above the earth. The range contribution from equations (14) is nearly 100 meters at the horizon.

The effects given by the above expressions are expected to be somewhat larger than the observed ones near the horizon because signal path curvature was neglected. The actual curved path, though geometrically slightly longer than the slant range vector, is "optically" shorter (cf. Fermat's principle).

An estimate of the curvature effects neglected in equations (12) and (14) has been obtained by tracing rays through sample atmospheres described by equations (7). Preliminary results show that neglect of path

bending in a nominal atmosphere may make the computed tropospheric contribution to the doppler shift or range rate 15 or even 20 percent too large at the geometrical horizon; but less than 3 percent too large at elevation angles above 5°. The neglected path curvature error in the range effect is approximately half as large as this. These effects are shown in Figure 7 for humid summer conditions; a fairly extreme case. More details will be published in a later paper. The presence of the curvature effect in low-angle doppler data is readily observable, as will be shown below.

B. Observed Results

Tropospheric effects can be observed (1) in the doppler residuals (observed minus theoretical doppler shift) obtained when no tropospheric correction has been applied to the observed data; and (2) in station position errors. Both types of observed effects will be shown here.

In processing satellite doppler data, the thoretical tropospheric correction is computed from equations (12) and (13) at each data point and applied to the data before final computations are made. Inputs are geometric conditions obtained from a preliminary orbit, station latitude for computing tropospheric height, and local surface weather data observed near the time of the satellite pass. Should weather data be lacking, seasonal values of N_{T_d} and N_{T_w} for the tracking station locality are used ["Monthly Climatic Data for the World;" also Bean et al, 1960, 1966].

Figures 8 to 11 show the effectiveness of the correction described above. Figures 8, 9 and 10 show the doppler residuals (observed minus theoretical doppler shift) for three satellite passes observed under different geometrical and climatic conditions: in each case (a) without and

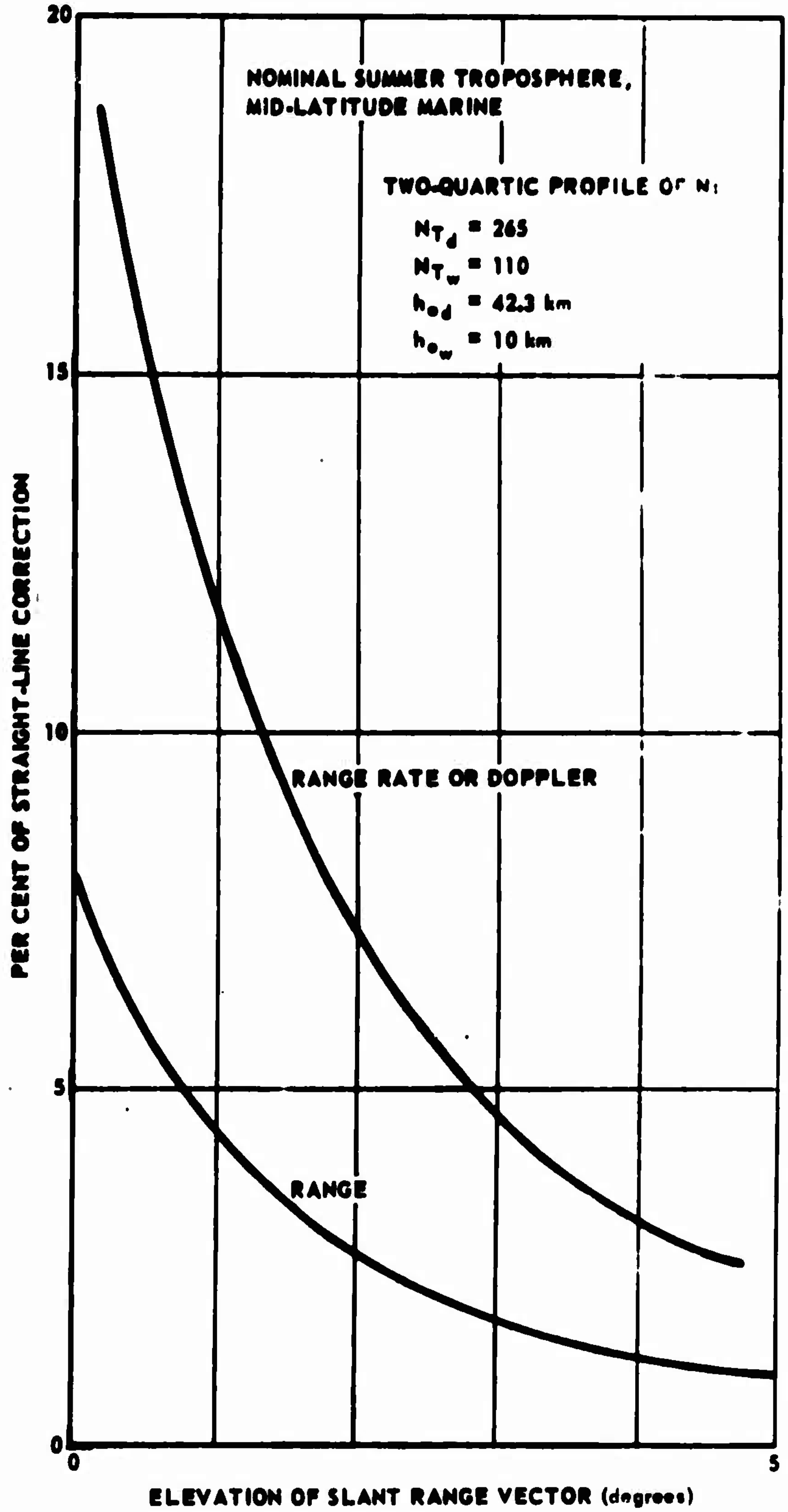

Fig. 7 ERROR IN TROPOSPHERIC CORRECTION (OVER-CORRECTION) WHEN SIGNAL PATH BENDING IS NEGLECTED

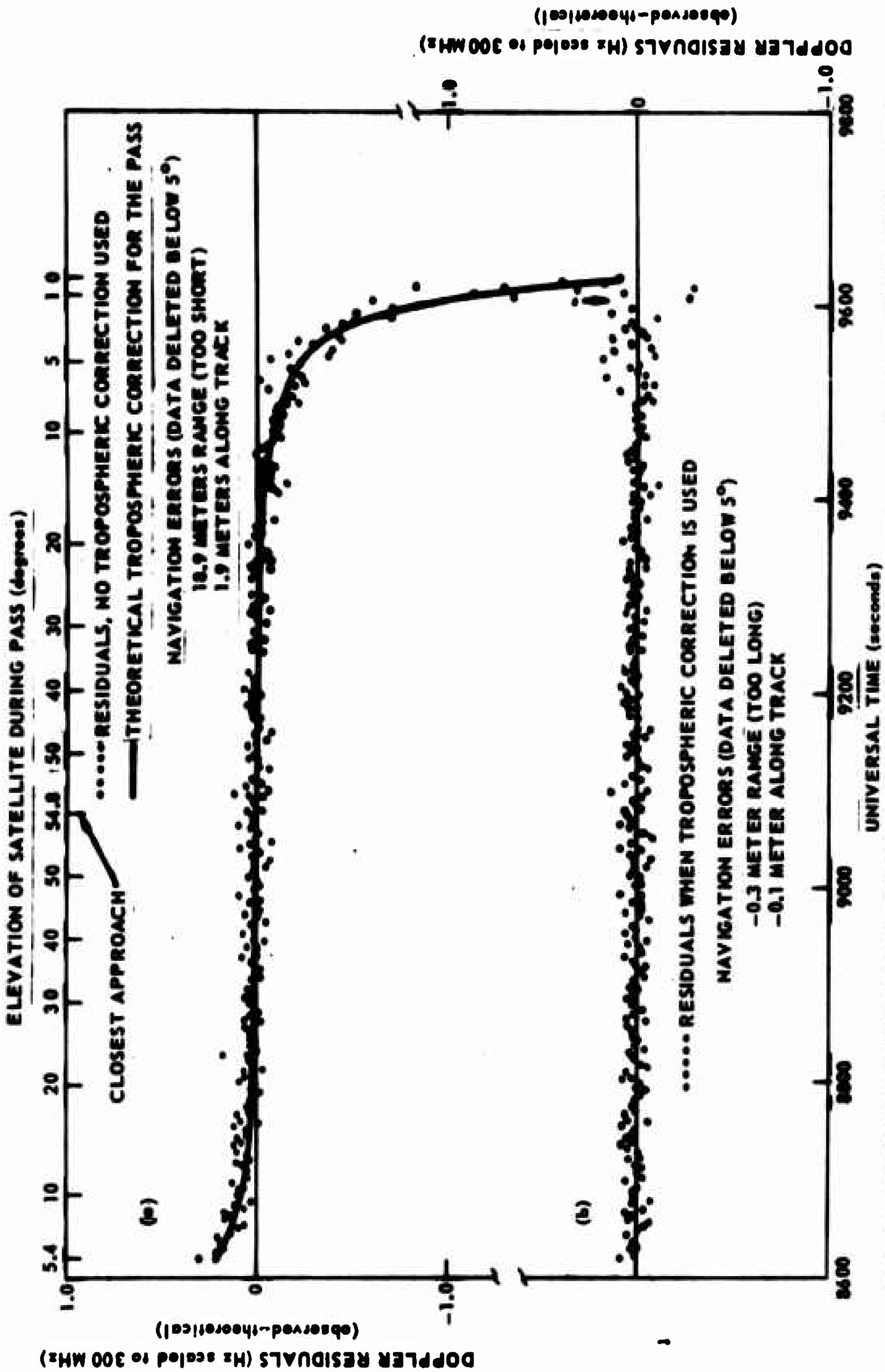

Fig. 8 TROPOSPHERIC EFFECT ON DOPPLER DATA DURING A SATELLITE PASS, SEYCHELLES ISLANDS, 2 SEPTEMBER 1967 – SATELLITE 1967 48A–SURFACE CONDITIONS: $N_{T_d} = 248$, $N_{T_w} = 101$ (STATION AT "TRUE" POSITION FOR COMPUTING RESIDUALS)

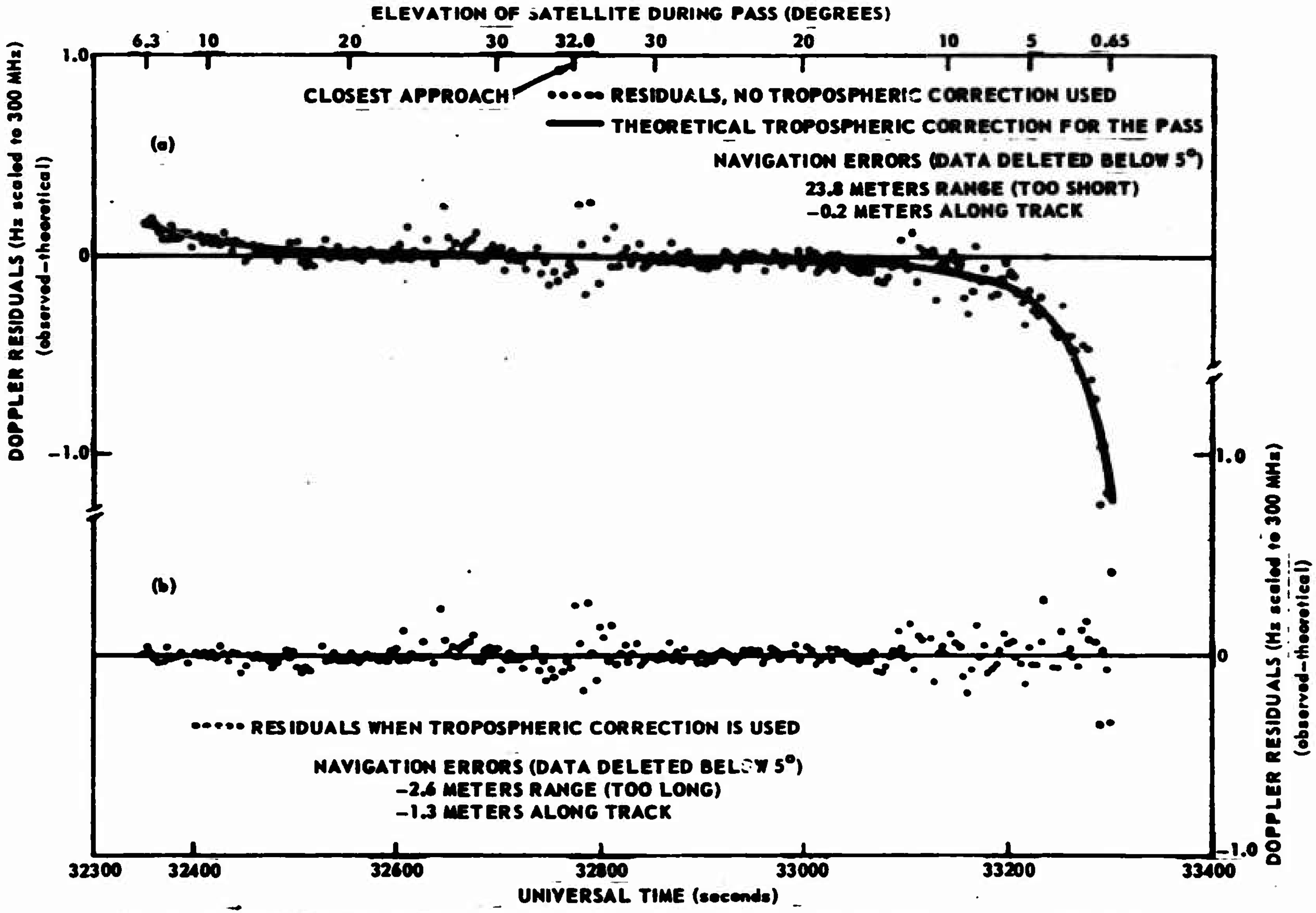

Fig. 9 TROPOSPHERIC EFFECT ON DOPPLER DATA DURING A SATELLITE PASS, McMURDO SOUND, ANTARCTICA, 3 JANUARY 1967 – SATELLITE 1965 48A–SURFACE CONDITIONS: $N_{T_d} = 284$, $N_{T_w} = 21$ (STATION AT "TRUE" POSITION FOR COMPUTING RESIDUALS)

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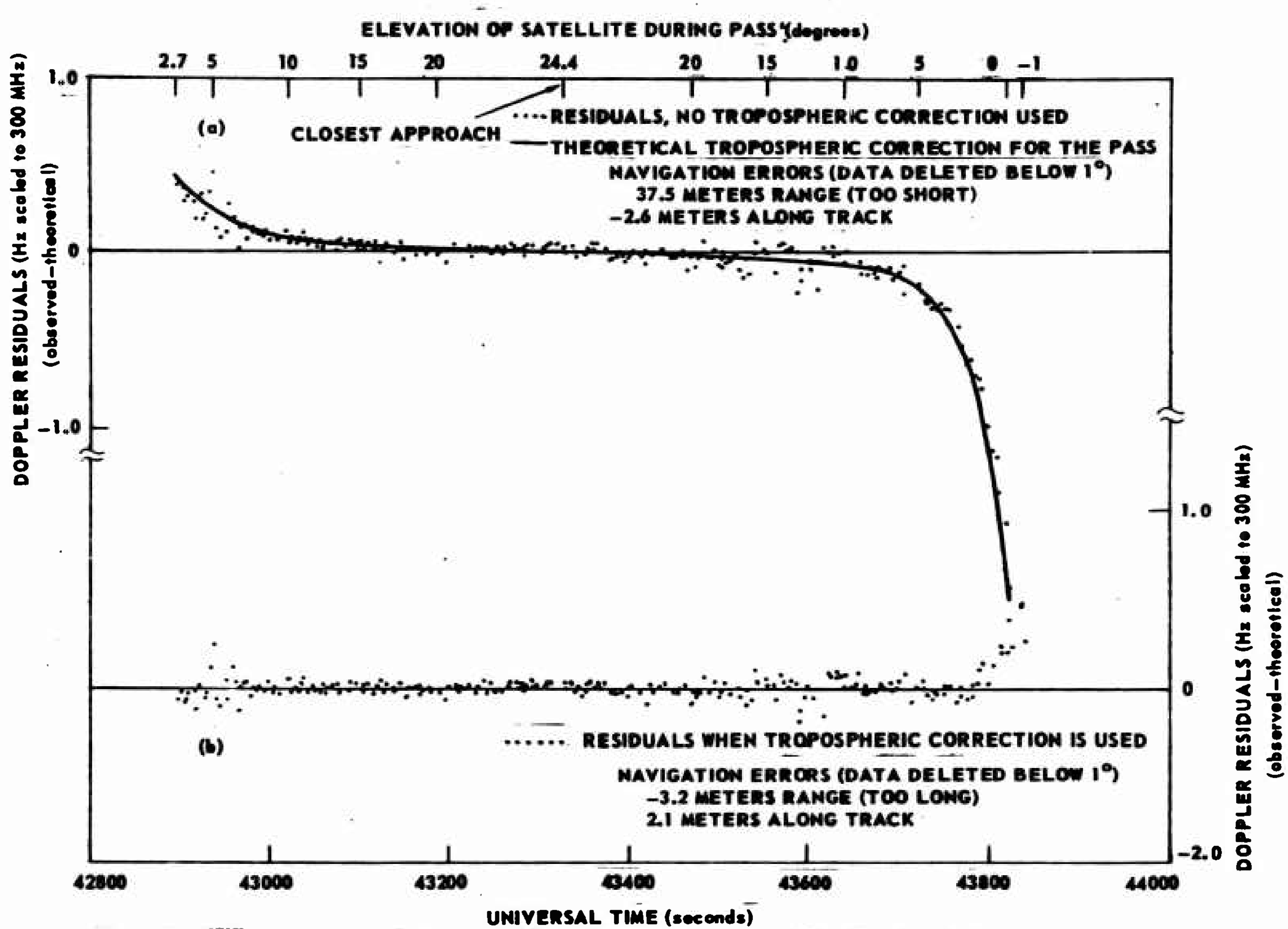

Fig. 10 TROPOSPHERIC EFFECT ON DOPPLER DATA DURING A SATELLITE PASS, LASHAM, ENGLAND, 2 SEPTEMBER 1967 – SATELLITE 1967 34A–SURFACE CONDITIONS: $N_{T_d} = 264$, $N_{T_w} = 55$ (STATION AT "TRUE" POSITION FOR COMPUTING RESIDUALS)

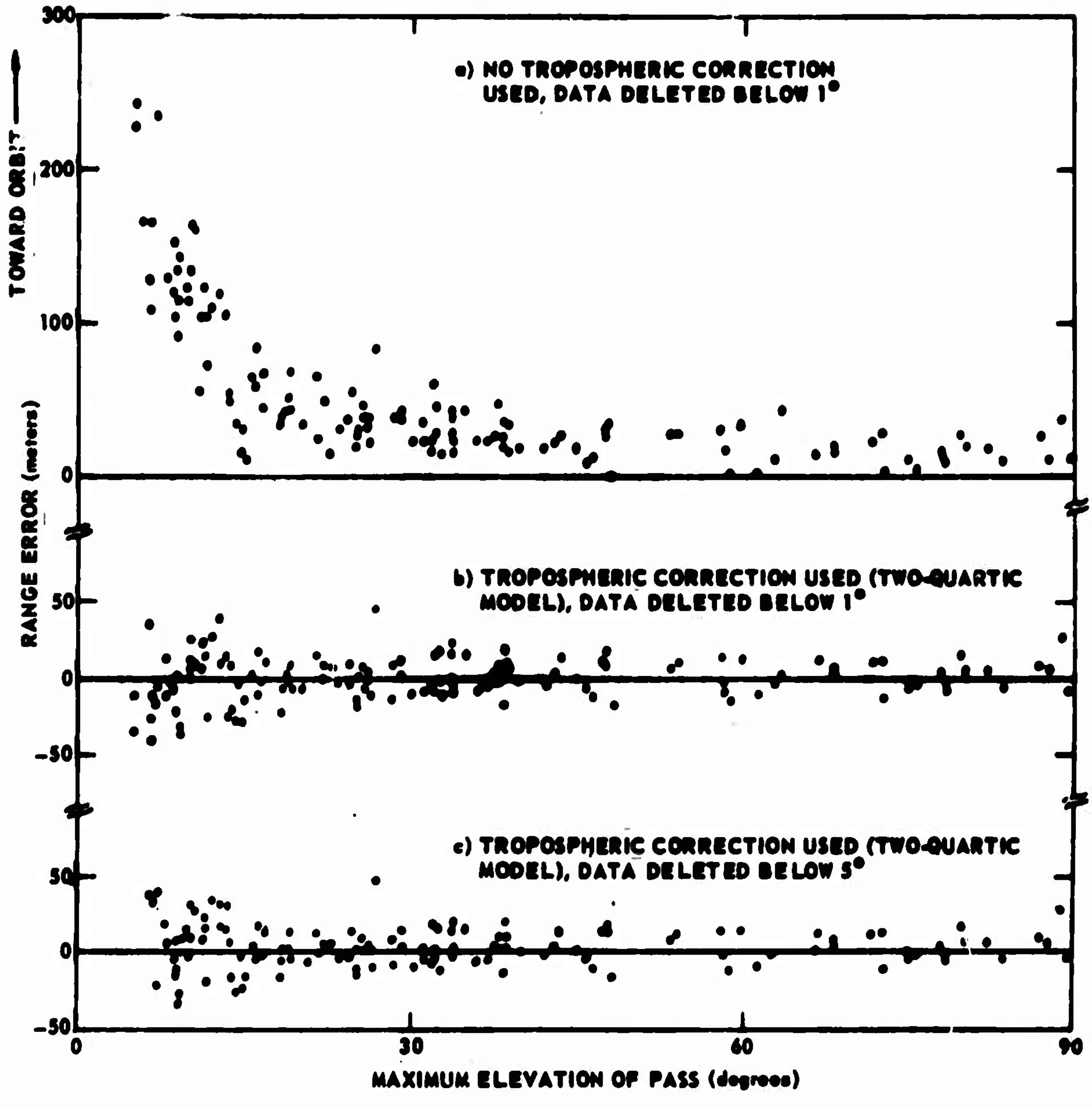

Fig. 11 NAVIGATION ERROR IN STATION-TO-ORBIT RANGE, SATELLITE 1967 34A, 1 AND 2 SEPTEMBER 1967

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(b) with the use of the tropospheric correction. The thoretical correction is also shown for each pass. In each figure the uncorrected residuals of part (a) follow the trend of the thoretical tropospheric effect. Little systematic error remained after the tropospheric correction had been applied (part (b) of each figure). In the one case where actual tracking extended below the geometrical horizon (Figure 10), the magnitude of the theoretical correction was noticeably too large below 2^{o} elevation. This is compatible with the expected but disregarded path curvature effect shown in Figure 7. The effect, however, is a function of local conditions and its magnitude would often be less than that of Figure 7.

It should be noted that Figures 8 to 10 show doppler residuals for passes in which non-tropospheric errors were very small. Other errors can easily mask tropospheric effects in the central portion of a pass, but even then, the characteristic large increase of the tropospheric effect near the horizon is routinely visible.

The effect of the troposphere on station-to-orbit range is shown in Figure 11. (Navigated station position in a direction parallel to the orbit is affected by the troposphere only if the tracking data are not symmetrical about the point of closest approach; and will not be discussed here.) Figure 11 shows, for passes at different elevation angles, the errors in station-to-orbit ranges ("navigated" ranges as compared with the best available theoretical values) for a two-day set of satellite passes, observed at tracking stations distributed over the earth. Each point represents one pass. Parts (a) and (b) show the range errors obtained from the

same tracking data, respectively without and with the use of the tropospheric correction; using all tracking data obtained down to an elevation angle of 1°.

Tropospheric effects, when uncorrected as in part (a) of Figure 11, shorten the apparent station-to-orbit range by 20 meters (average) in the case of high-angle passes, but by very much more for lower passes (40 meters for a pass whose maximum elevation is 30°, 100 meters for a 10° pass, and more than that for still lower passes).

These systematic errors were removed by the use of the tropospheric correction, as shown in part (b). For passes lower than 10°, however, the average correction was a little too large. This over-correction can reasonably be ascribed to the (neglected) low-angle path curvature effect which has already been pointed out in the doppler residuals.

The over-correction disappeared when data were deleted below an elevation of 5°, as in Figure 11(c). Here no appreciable systematic effect remains. The correction was satisfactory on the average, and the neglected effect of signal path curvature was apparently unimportant as long as the satellite elevation during the data interval was at least 5°.

VI. Discussion

The residual range errors of Figure 11(c) show little or no bias as a group. Their scatter has many possible sources, including uncorrected tropospheric effects. Such effects may produce an error in the range determination which is systematic for any one satellite pass. The fact

that the scatter is noticeably greater for the lower-angle passes indicates the atmosphere as one probable source.

On the basis of probable errors in the inputs and assumptions used above, it is estimated that for any one pass, tropospheric effects in doppler data are reduced at least by an order of magnitude by the correction given above. A few remarks will now be made about the remaining errors, and about some areas for further investigation; in particular, height parameters, and local weather effects.

The area under any N_w profile, even for a humid atmosphere, represents a small fraction (seldom more than 10 percent) of the area under the total N profile (Figures 1 to 4); and contributes only this same fraction to the total tropospheric effect (range or doppler), in the zenith direction. The same rule holds approximately, down to about 5^{o} elevation, but not below. At the geometrical horizon, the moisture content of the air may account for as much as 25 or 30 percent of the total tropospheric doppler effect (from equation (12)). If data below 5^{o} elevation are deleted, however, the "dry" portion of N accounts for 90 percent or more of the tropospheric range or doppler effect.

Thus an error of 20 percent in representing the N_w profile area changes the total tropospheric effect by 2 percent or less, while an error of only 2 percent in the N_d profile area changes the total effect by nearly 2 percent. The relatively minor importance of the "wet" term, for satellite range or doppler applications, which was not recognized when the Model I troposphere correction was developed, became obvious when the two profile components were studied separately. For future improvement, the height parameters, especially for the "dry" term, should be better determined

as to latitude variation, local climatic characteristics, and possibly time variation (e.g., seasonal or diurnal).

An illustration of local weather effects is presented in Figures 12 and 13: two sample N profiles obtained from individual balloon ascents at Pensacola, Florida, six days apart; with their companion theoretical profiles. The theoretical "dry" height was routinely obtained from equation (10), the assumed "wet" height being 12 km. The observed "dry" component profiles are relatively smooth and similar in shape, but there is a marked difference between the "wet" components for the two days. These "wet" profiles can be qualitatively correlated with weather maps for the dates. A cold front had reached the location shortly before the Figure 12 observations. The arriving air mass was "continental polar"; however, this had not yet reached very high, as can be seen by the increase of N_w with height, just above the surface. At the time of the Figure 13 observations, there had been no recent weather front and the air mass in the region was "maritime tropical" and showed a steep decrease of N_w with height. Minor irregularities in the "wet" terms occurred at higher altitudes on both days. In spite of the great difference in the "wet" profiles, the observed value of the zenith integral $\int Ndh$ (total) was only one percent higher for Figure 13 than for Figure 12. A two-quartic correction based on surface data would have been 3 percent higher for the Figure 13 than for the Figure 12 data, leaving a 2 percent differential in the precision of the correction for the two days.

Aside from temporary weather variations and systematic latitude variations, other climatic differences may exist for which some systematic allowance could be made. Temperature inversions, for example, are so

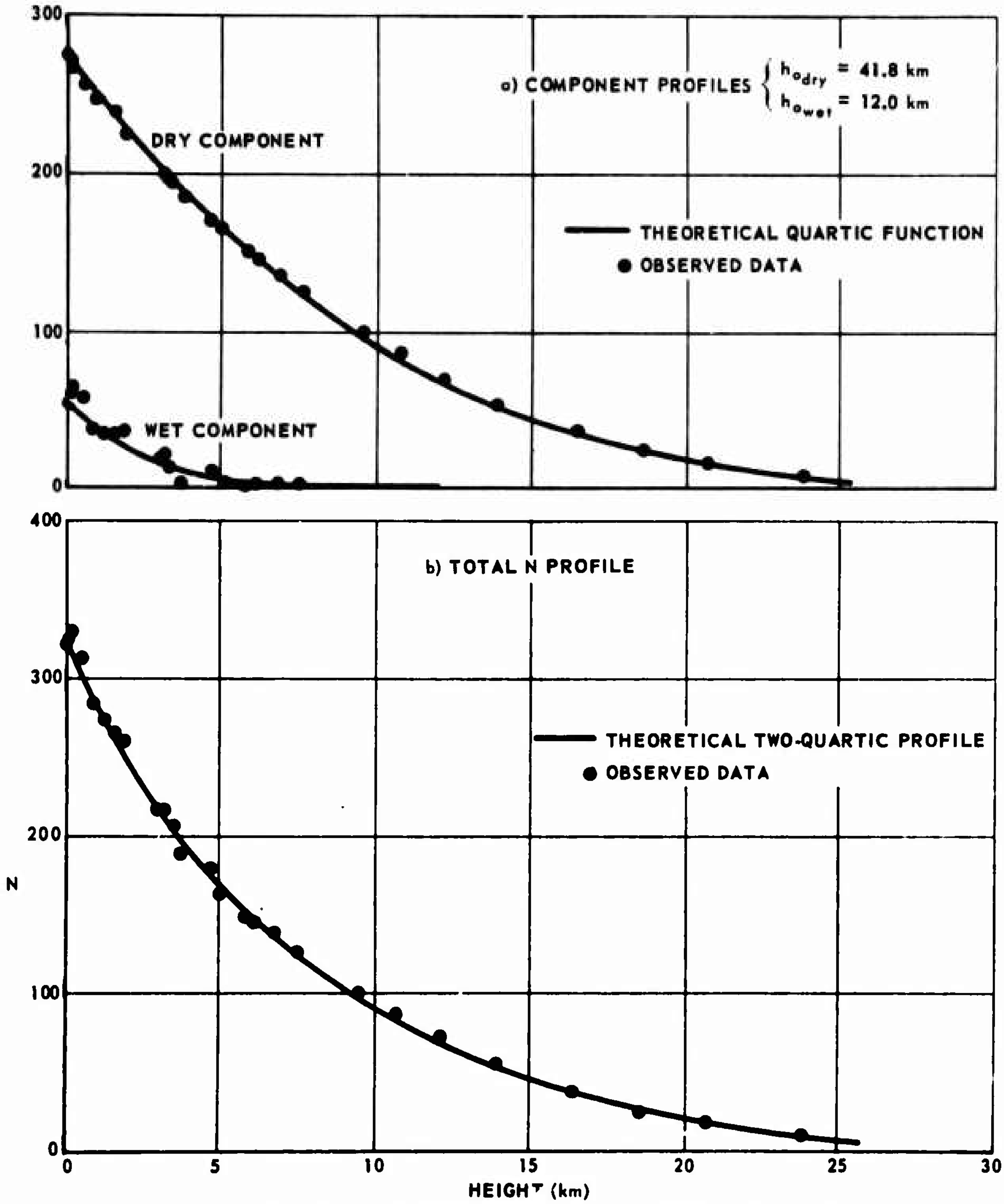

Fig. 12 REFRACTIVITY PROFILE, PENSACOLA, FLORIDA
21 APRIL 1965, 12 HOURS U.T.

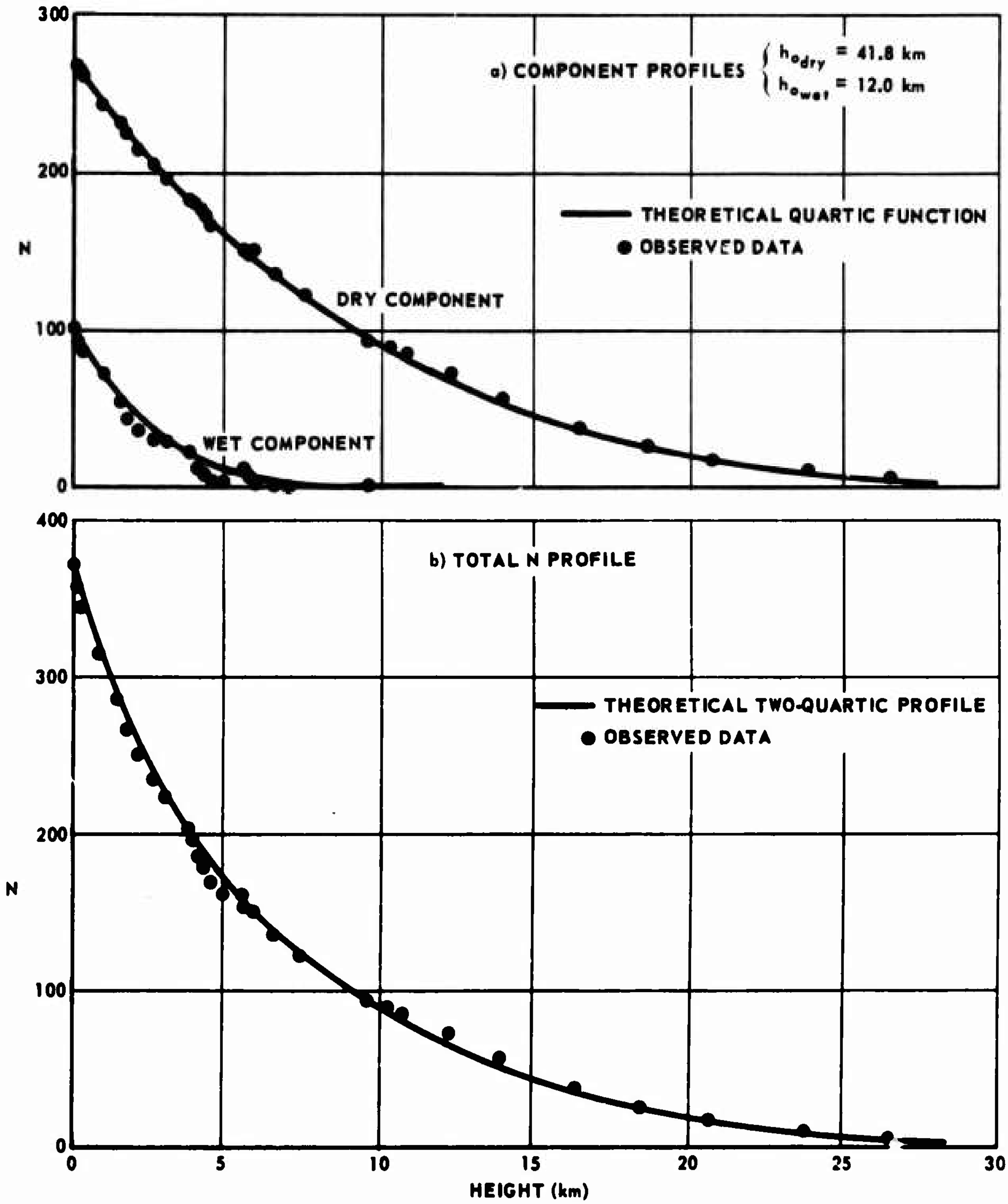

Fig. 13 REFRACTIVITY PROFILE, PENSACOLA, FLORIDA
27 APRIL 1965, 12 HOURS U.T.

common as to be expected in some areas and seasons. If the optimum correction is desired at a specific tracking station, the N profile in the region should be sufficiently observed to determine its systematic and predictable variations from the nominal profile.

The N profile presented above may be of interest also for correcting optical data [Guier, 1968], since the "dry" term of the radio refractivity is very nearly the same as the optical refractivity of air. The quartic "dry" component profile could be used with little modification for laser and other optical applications.

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